Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 5, pp. 872–877, 2001

CALCULATION TECHNIQUE FOR AEROELASTIC OSCILLATIONS OF MULTIBEAM CONSTRUCTIONS

S. D. Salenko

UDC 533.6.013.42

Mathematical models and techniques that describe transverse oscillations of constructions in a wind flow are considered. A technique is proposed, which allows one to determine the amplitudes of oscillations of multibeam constructions under wind resonance. Oscillations of a system with distributed parameters are described by a weakly nonlinear differential equation. The amplitudes of the limiting cycles of oscillations are determined using the energy approach. To describe the fluctuating component of aerodynamic forces, an empirical coefficient is introduced, which characterizes the energy contribution of these forces per one period of oscillations. In their structure, the calculation formulas are similar to those in European standards for calculating the response of constructions under the action of wind, but the formulas proposed are more universal and describe the physics of the process more adequately.

The following main types of aeroelastic oscillations are currently distinguished: flutter, wind resonance, galloping, and buffeting [1–3]. The mathematical models and techniques for the description of transverse oscillations of constructions in a wind flow were mostly derived for single-beam constructions.

In modern bridge-building, the method of longitudinal sliding is used to place span constructions in a design position. To lighten the cantilever, overlapping plates are taken off at the front part of the construction; as a result, it is a construction consisting of several box girders located at a distance of 3–6 heights and connected by transverse links in the form of trusses (Fig. 1). It is known from experience of constructing this kind of bridges [4] and wind-tunnel experiments with span-construction models [5] that a section of a span construction without orthotropic plates generates significant oscillating aerodynamic loads. The calculations of the shape and frequency of oscillations of a system of beams connected by lintels showed that crossflow flexural oscillations in the vertical plane are of practical interest for this class of constructions. Other shapes of oscillations may appear only under the action of unrealistically high velocities of the wind.

It follows from the results of wind-tunnel experiments with models of two- and three-beam span constructions that there are two or three resonant velocities of the wind at which intense crossflow oscillations of the construction are observed in the case of multibeam constructions, in contrast to single beams [5]. The existence of several critical Strouhal numbers is caused by the possibility of existence of different types of the flow around multibeam constructions; oscillations in different regimes are significantly different in terms of the character of excitation (rigid or soft), width of the range they cover, and amplitude. Within the framework of the currently existing classification, these oscillations are most close to wind resonance, though at least one of the resonance modes is related to aerodynamic interference of the beams.

Several basic approaches may be classified, which are used to describe transverse oscillations of constructions exposed to a flow. In the simplest formulation, the system is considered as a linear oscillator, and the aerodynamic force is assumed to vary in accordance with a harmonic law [1, 6]. The aerodynamic force is often represented as a sum of two components, which coincide in phase with displacement and velocity of the body [2, 7]; sometimes these two approaches are united [8]. Not only determinate but also statistical approaches are used to describe transverse

Novosibirsk State Technical University, Novosibirsk 630092. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 5, pp. 161–167, September–October, 2001. Original article submitted January 29, 2001; revision submitted March 26, 2001.

 0021-8944/01/4205-0872 $25.00

Fig. 1. Three-beam cantilever of the span construction.

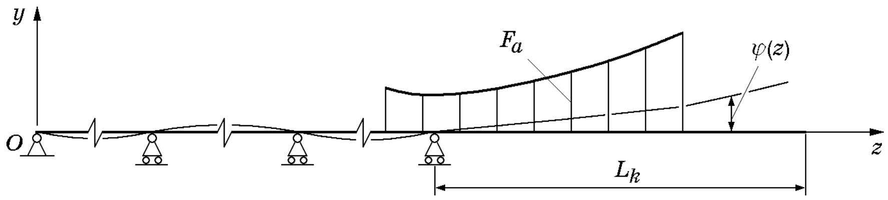

Fig. 2. Calculation scheme of the span-construction cantilever.

oscillations [9]. A technique based on similarity of the behavior of the oscillating system considered and the Van-der-Pol oscillator is widely used [10, 11]. These techniques for single beams allow reproduction of the amplitude of oscillations, the width of the resonance region, and the hysteresis phenomena. Nevertheless, on the one hand, these techniques utilize a complex mathematical apparatus and are inconvenient for engineering calculations, and on the other hand, being semiempirical, they do not allow one to predict the behavior of new constructions, since they require a large number of experimental data.

The objective of the present work is to develop a technique that allows one to determine the resonant velocities and amplitudes of the limiting cycles of aeroelastic oscillations of multibeam constructions with a minimum amount of input experimental data.

The constructions considered are multisupport beams of mass and rigidity varied over the length (Fig. 2). To describe flexural oscillations in the vertical plane, we use the following assumptions: friction in supports is ignored and they are assumed to be absolutely rigid; the beam is assumed to be long, i.e., the relation $l/h \gg 1$ is valid [l is the beam length and h is its characteristic size (cross-sectional height)]; the cutting forces are neglected, since $l/h \gg 1$.

The differential equation of transverse oscillations of the beam under wind resonance is obtained by introducing terms that characterize aerodynamic forces and internal scattering of energy in the process of oscillations into the equation for free oscillations:

$$\frac{\partial^2}{\partial z^2}\Big(EI(z)\,\frac{\partial^2 w}{\partial z^2}\Big) + m(z)\,\frac{\partial^2 w}{\partial t^2} = F_a(z,t) + \frac{\partial^2}{\partial z^2}\Big[\Phi\Big(\frac{\partial^2 w}{\partial z^2}\Big)\Big]. \qquad (1)$$

Here $w(z,t)$ is the function of flexural deformation, $m(z)$ is the running mass, $EI(z)$ is the flexural rigidity, and F_a is the fluctuating component of the running aerodynamic load.

In the general case, F_a depends on the local amplitude of oscillations, Reynolds and Strouhal numbers, interference of the neighboring sections, and tip effects. We assume that, for steady oscillations, F_a is a periodic functions with a period equal to the period of oscillations of the beam.

To describe the internal scattering of energy in the process of oscillations, we introduce the functional $\Phi(\partial^2 w/\partial z^2)$, which characterizes the imperfect elasticity of the oscillating system and which is the moment of forces of inelastic resistance [12].

It follows from the test results for span-construction models that the logarithmic decrement of oscillations without the flow is 0.01–0.02, and the increment of oscillations developing in the flow is of the same order. Therefore, we may assume that the terms F_a and $\partial^2[\Phi(\partial^2 w/\partial z^2)]/\partial z^2$ in Eq. (1) have a higher order of smallness than the terms in the left-hand side; therefore, Eq. (1) may be considered as weakly nonlinear.

Taking into account the smallness of the right-hand side of Eq. (1), we may assume that beam oscillations in a steady regime obey a harmonic law with a frequency almost equal to the frequency of eigenoscillations. Then, we may assume that $w(z,t) \approx a\varphi(z)\sin\omega t$, where a is the amplitude of steady oscillations, $\varphi(z)$ is the own shape of oscillations, and ω is the eigenfrequency of oscillations.

To determine the amplitudes of the limiting cycles of self-induced oscillations, we use the principle of energy balance, according to which energy scattering in an oscillating system in a cycle ΔW equals the work of external forces ΔA: $\Delta W = \Delta A$.

Integrating the expression for the elementary work of external aerodynamic forces at an elementary section with respect to the period and length of the beam, we obtain

$$\Delta A = \int_0^{L_q}\int_0^{2\pi} F_a \omega a \varphi(z)\cos\omega t\,dt\,dz. \qquad (2)$$

Since F_a is a periodic function in the case of steady oscillations, it can be expanded into the Fourier series $F_a = \sum_{n=1}^{\infty} q_n \sin(n\omega t + \psi_n)$, where $q_n = q_n(t, a, \ldots)$ are the amplitudes of oscillations of the distributed aerodynamic load at the nth frequency and ψ_n is the phase difference between the nth harmonic and beam displacement. We consider oscillations of the first tone, though similar reasoning may be applied to an arbitrary tone. Taking into account that the resulting work of harmonics with $n > 1$ over the period equals zero, Eq. (2) yields

$$\Delta A = a\pi \int_{L_q} q_1 \varphi(z)\sin\psi_1\,dz = a\pi \int_{L_q} c'_{y1}\,\frac{\rho V^2}{2}\,b\varphi(z)\sin\psi_1\,dz, \qquad (3)$$

where c'_{y1} is the amplitude of the fluctuating component of the lift coefficient, b is the overall width of the beams in the streamwise direction, and ρ and V are the free-stream density and velocity, respectively.

Assuming that $\psi_1 \approx 90°$ in Eq. (3) (which is the case for the usual resonance), we obtain the same result as for a harmonic external force independent of the character of oscillations. However, in the case of aeroelastic oscillations considered, the external aerodynamic force is related to the character of oscillations. Experiments with prisms of rectangular cross sections show [13] that the phase difference ψ_1 varies within a wide range; it is difficult to determine experimentally the value of ψ_1 and to systematize and approximate it because of its complex dependence on the cross-sectional shape of the prisms, the amplitude of oscillations, and the Strouhal number Sh.

It seems reasonable not to separate the quantities c'_{y1} and $\sin\psi_1$ but to introduce the coefficient $c_a = c'_{y1}\sin\psi_1$, which is a functional characterizing the work of aerodynamic forces during one period for the tone considered. In the general case, the coefficient c_a may be either positive (excitation of oscillations) or negative (damping of oscillations). The coefficient c_a is determined experimentally. The dependence of the coefficient c_a on the relative amplitude of oscillations for three resonant regions (three values of Sh number) for a three-beam section model is plotted in Fig. 3. For a section model, the amplitude of oscillations over the length is constant along the streamwise coordinate. For actual constructions, the local amplitude depends on the longitudinal coordinate z. Because of the interference of the neighboring sections with different amplitudes of oscillations, the problem of determination of the local value of the coefficient c_a arises. By analogy with the hypothesis of planar sections for a wing, we assume that the local value of the coefficient c_a depends only on the local relative amplitude of oscillations: $c_a = c_a(a\varphi(z)/h)$. Since the experimental dependences $c_a = c_a(a/h)$ are approximated at the section $a/h \geqslant 0$, and the amplitude may have an arbitrary sign in calculations, the absolute value of $\varphi(z)$ should be used in (3).

In addition, we introduce the coefficient $R(z, a/h)$ into (3), which takes into account the correlation of oscillations of the aerodynamic force along the construction. We approximate the coefficient $R(z, a/h)$ by the formula obtained on the basis of experimental data of [8, 13]: $R(z,a) = 1 + \exp(-15\bar{a}^2)[\exp(-(\bar{z}/5)^2) - 1]$, where $\bar{a}$ and $\bar{z}$ are the amplitude of oscillations and the streamwise coordinate normalized to the beam height h.

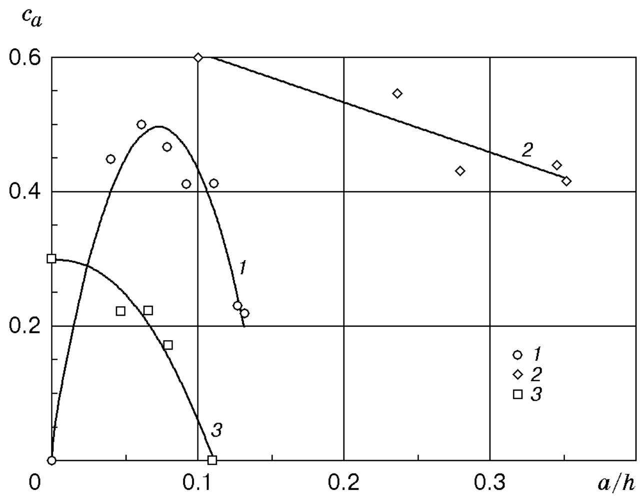

Fig. 3. Coefficient c_a as a function of the relative amplitude for Sh = 0.127 (1), 0.087 (2), and 0.084 (3).

Taking into account the above reasoning, we obtain the following expression from Eq. (3):

$$\Delta A(a_{\max}) = a_{\max}\pi \frac{\rho V^2}{2} b \int\limits_{L_q} c_a R(z,a)|\varphi(z)|\,dz \tag{4}$$

($a_{\max}$ is the maximum amplitude of oscillations).

The magnitude of internal scattering of energy in one period ΔW is determined by the known technique [12]:

$$\Delta W(a_{\max}) = \delta\omega^2 a_{\max}^2 \Big[\sum_j \int\limits_{L_j} m(z)\varphi^2(z)\,dz + \sum_i m_i\varphi_i^2(z)\Big]. \tag{5}$$

Here δ is the logarithmic decrement of oscillations, m_i are the masses of localized loads, and φ_i is the amplitude value of flexure at the place of the ith localized load.

Since we assumed that $\Delta A = \Delta W$, then equating (4) and (5), we obtain the expression for determining the maximum amplitude of steady oscillations

$$a_{\max} = \pi \frac{\rho V^2}{2} b \int\limits_{L_q} c_a R(z,a)|\varphi(z)|\,dz \Big/ \Big[\delta(a)\omega^2\Big(\sum_j \int\limits_{L_j} m(z)\varphi^2(z)\,dz + \sum_i m_i\varphi_i^2(z)\Big)\Big]. \tag{6}$$

Formula (6) is similar to that in [14] and European standards for calculating constructions [15]:

$$\frac{y_0}{d} = \bar{K}K\,\frac{c_{\text{lift}}}{\text{Sc}\,\text{Sh}^2}, \tag{7}$$

where c_{lift} is the coefficient of the exciting force, $\bar{K}$ is the coefficient characterizing the shape of oscillations, K is the correction coefficient:

$$\bar{K} = \sum_j \int\limits_{L_j} |\varphi(z)|\,dz \Big/ \Big(4\pi \sum_j \int\limits_{L_j} \varphi^2(z)\,dz\Big), \qquad K = \int\limits_{L_q} |\varphi(z)|\,dz \Big/ \sum_j \int\limits_{L_j} |\varphi(z)|\,dz;$$

y_0 is the maximum displacement of the construction (amplitude of oscillations), d is the characteristic size, Sc is the Scruton number, and Sh is the Strouhal number.

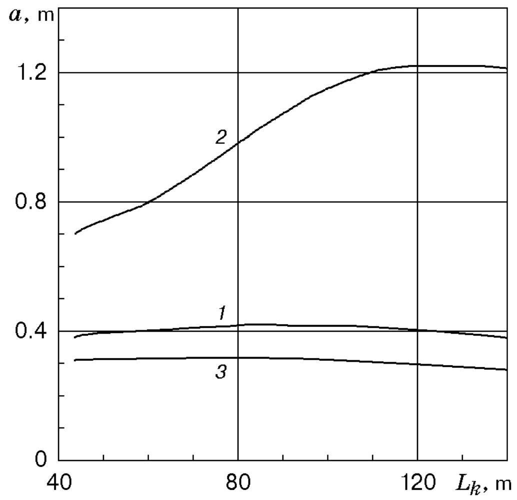

Fig. 4. Amplitudes of oscillations of the span-construction cantilever for Sh = 0.127 (1), 0.087 (2), and 0.084 (3).

Indeed, we obtain the following relation from Eq. (6) in the absence of localized loads:

$$\bar{a} = \frac{a}{h} = \pi \frac{\rho V^2}{2} b \int\limits_{L_q} c_a R(z,a)|\varphi(z)|\,dz \Big/ \Big(h\delta\omega^2 \sum_j \int\limits_{L_j} m(z)\varphi^2(z)\,dz\Big)$$

$$= \pi\rho V^2 b \int\limits_{L_q} c_a R(z,a)|\varphi(z)|\,dz \Big/ \Big(2h\delta 4\pi^2 f^2 \sum_j \int\limits_{L_j} m(z)\varphi^2(z)\,dz\Big)$$

$$= \Big[\rho h^2 \sum_j \int\limits_{L_j} \varphi^2(z)\,dz \Big/ \Big(2\delta \sum_j \int\limits_{L_j} m(z)\varphi^2(z)\,dz\Big)\Big] \frac{V^2}{f^2 h^2} \Big[\sum_j \int\limits_{L_j} |\varphi(z)|\,dz$$

$$\Big/ \Big(4\pi \sum_j \int\limits_{L_j} \varphi^2(z)\,dz\Big)\Big] \Big[b \int\limits_{L_q} c_a R(z,a)|\varphi(z)|\,dz \Big/ \Big(h \sum_j \int\limits_{L_j} |\varphi(z)|\,dz\Big)\Big] = \bar{K}\,\frac{1}{\mathrm{Sc}\,\mathrm{Sh}^2}\,\frac{b}{h}\,\tilde{c}_a. \qquad (8)$$

Here $\tilde{c}_a = \int\limits_{L_q} c_a R(z,a)|\varphi(z)|\,dz$ is the generalized (equivalent) coefficient c_a for the beam as a whole.

Equation (8) contains the ratio b/h, which is absent in (7). Equation (7) is obtained for a cylinder with a characteristic size d, which enters both the expression for the coefficient of the exciting force and the Strouhal number. Nevertheless, in the case of a prismatic body, it is more convenient to use different characteristic dimensions for the above two quantities. For c_a, this is the total cross-sectional width b (since it determines the area affected by pressure fluctuations that excite oscillations); for Sh, this is the cross-sectional height h (since it mainly determines the vortex-shedding frequency). Using this approach, it is more convenient to generalize and systematize experimental dependences for various cross-sectional shapes.

In addition, Eq. (7) contains c_{lift}, which is the amplitude of the fluctuating component of the lift coefficient. The simplicity of this approach (where the system is considered as a linear oscillator) was discussed above.

It should be noted that the European standards for calculating constructions imply that the coefficient c_{lift} and the Scruton number (or the logarithmic decrement of oscillations δ) are independent of the amplitude of oscillations, which is not usually fulfilled.

Let c_0 be the coefficient at the zeroth power in the expansion of c_a into the Maclaurin series $c_a = \sum\limits_n c_i \bar{a}^n$ [for small amplitudes, $c_a \approx c_0 + (c_a)'_{\bar{a}}\bar{a}$]. From an analysis of the variants of the solution of Eq. (6), it follows that the following cases are possible:

— for $c_0 > 0$ (curve 3 in Fig. 3), steady self-induced oscillations will be observed for all δ;

— for $c_0 = 0$ (curve 1 in Fig. 3) and the logarithmic decrement of oscillations at zero amplitude δ_0 greater than some critical value

$$\delta_0 > \delta_{\rm cr} = (c_a)'_{\bar a}\pi\,\frac{\rho V^2}{2}\,b\int\limits_{L_q} R(z,a)|\varphi(z)|\,dz\Big/\Big[\omega^2\Big(\sum_j\int\limits_{L_j} m(z)\varphi^2(z)\,dz + \sum_i m_i\varphi_i^2(z)\Big)\Big],$$

oscillations will not occur (or will decay for finite perturbations), which is supported by both experimental and numerical data;

— if there is rigid excitation of oscillations (curve 2 in Fig. 3) in a given cross section of beams on the section model, the oscillations on the cantilever beam will either reach a finite amplitude only at a certain initial impact or decay at a high logarithmic decrement of oscillations, whose magnitude is determined in calculations by formula (6) using the method of successive approximations.

The calculated dependences of the amplitude of oscillations on the overhang (length of the cantilever section) L_k for an actual span construction of the bridge are plotted in Fig. 4. The maximum amplitudes of oscillations are reached at a certain intermediate value of the overhang rather than at its maximum value (as it is often expected). There are no full-scale measurements for such constructions, except for the visually observed oscillations with an amplitude of 0.3–0.5 m for a 80-m overhang of a three-beam cantilever [4].

The technique proposed was experimentally verified in a wind tunnel using a model whose dynamic characteristics are similar to characteristics of the span construction of the bridge. The amplitudes of oscillations calculated using the coefficients c_a obtained for the section model are in good agreement with the measured values (the error is less than 5–10%).

REFERENCES

1. E. Simiu and R. Scanlan, *Action of Wind on Buildings and Constructions* [Russian translation], Stroiizdat, Moscow (1984).
2. G. M. Fomin, "Self-induced oscillations of elastic constructions in flow stalling," Doct. Dissertation in Tech. Sci., Moscow (1974).
3. M. I. Kazakevich, *Aerodynamics of Bridges* [in Russian], Transport, Moscow (1987).
4. S. D. Salenko, A. A. Kuraev, A. D. Obukhovskii, et al., "Quenchers of aeroelastic oscillations of a span-construction cantilever," in: *Construction and Reconstruction of Railway and Highway Bridges* (collected scientific paper) [in Russian], No. 1, 53–58 (1996).
5. S. D. Salenko and A. D. Obukhovsky, "Aeroelastic oscillation especiality of multibeam consols," in: *Proc. of the ICMAR'96* (Novosibirsk, Sept. 2–6, 1996), Inst. Theor. and Appl. Mech., Sib. Div., USSR Acad. of Sci., Novosibirsk (1996), pp. 205–209.
6. G. V. Fershing, *Fundamentals of Aeroelasticity* [Russian translation], Mashinostroenie, Moscow (1984).
7. C. Scruton and A. R. Flint, "Wind-excited oscillations of structures," *Proc. Inst. Civ. Eng.* (*London*), **27**, 673–702 (1964).
8. B. J. Vickery and R. I. Basu, "Across-wind vibrations of structures of circular cross-section. Part 1. Development of a mathematical model for two-dimensional conditions," *J. Wind Eng. Industr. Aerodyn.*, **12**, No. 1, 49–73 (1983).
9. R. W. Blevins and T. E. Burton, "Fluid forces induced by vortex shedding," in: *Proc. of Am. Soc. Eng.-Mech. Theor. Principles of Eng. Calculations*: Transl. J. *Trans. ASME, J. Basic Eng.*, No. 1 (1976), pp. 125–134.
10. R. A. Scop and O. M. Griffin, "On a theory for the vortex-excited oscillations of flexible cylindrical structures," *J. Sound Vibr.*, **41**, 263–274 (1974).
11. R. Landl, "A mathematical model for the vortex-excited vibrations of bluff bodies," *J. Sound Vibr.*, **42**, 219–234 (1975).
12. G. S. Pisarenko, *Generalized Nonlinear Model Taking into Account Energy Scattering During Oscillations* [in Russian], Naukova Dumka, Kiev (1985).
13. P. W. Bearman and E. D. Obasaju, "An experimental study of pressure fluctuations on fixed and oscillating square-section cylinders," *J. Fluid Mech.*, **119**, 297–321 (1982).
14. H. Ruscheweyh, M. Hortmans, and C. Schnakenberg, "Vortex-excited vibrations and galloping of slender elements," in: *Proc. of the East Europ. Conf. on Wind Eng.* (EECWE'94) (Warsaw, Poland, July 4–8, 1994), Vol. 1, Part 2, Warsaw (1994), pp. 189–194.
15. *Eurocode 1: Basis Design and Action on Structures*, Parts 2–4: *Wind Action*, Europ. Comm. Standard, Centr. Secretariat, Brussels (1994).